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数理解析研究所講究録
1037巻 1998年 86-96

Hypersurface purely elliptic singularities of (0,1)-type

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Introduction.

Let (X,x) be a germ of an normal isolated singularity of an analytic space. X often denote a sufficiently small Stein neighbourhood of the germ (X,x) of the singularity.

Watanebe introduced plurigenera $\{\delta_m\}_{m\in\mathbf{N}}$ for normal isolated singularities(see Def.1.1) and defined a purely elliptic singularity (see Def.1.2). If (X,x) is a two-dimensional purely elliptic singularity, (X,x) is a Gorenstein singularity. But if the dimension of (X,x) is greater than two, (X,x) is not always a Gorenstein singularity.

Ishii [2] investigated the structures and the dual graphs of quasi-Gorenstein purely elliptic singularities using the theory of mixed Hodge structure and classified them into 2n classes(see Def.1.3), including the condition that the singularity is Gorenstein or not. The result of Ishii's classification is shown in Table 1 and in Table 2 when n=2 and 3.

A two-dimensional purely elliptic singularity of $(0,0)$-type or $(0,1)$-type is known as a simple elliptic singularity or a cusp singilarity, respectively. Originally, a simple elliptic singularity (X,x) is defined as a singularity such that the exceptional set of the minimal resolution of (X,x) is a nonsingular elliptic curve, and a cusp singularity (X,x) is defined as a singularity such that the exceptional set of the minimal resolution of (X,x) is a circle of rational curves. Simple elliptic singularities are studied by Saito [6] in detail. In case

	Gorenstein
$(0,0)$	a circle of rational curves
$(0,1)$	an elliptic curve

Table 1: In two-dim. case.

	Gorenstein	Non-Gorenstein
$(0,0)$	a triangulation of S_2	a triangulation of T_2
$(0,1)$	a chain of surfaces $E_1, E_2, \cdots, E_r$	a circle of elliptic ruled surfaces
$(0,2)$	a $K3$ surface	an Abelian surface

Table 2: In three-dim. case.

that the self-intersection number of the elliptic curve E appearing as the exceptional set is -1, -2, or -3, (X,x) is a hypersurface singularity, and the defining equation of (X,x) is given by one of the following in some coordinates z_1, z_2, and z_3 around x:

$$\tilde{E}_6 : z_1^3 + z_2^3 + z_3^3 + \lambda_1 z_1 z_2 z_3 = 0 \; (E \cdot E = -3);$$
$$\tilde{E}_7 : z_1^2 + z_2^4 + z_3^4 + \lambda_2 z_1 z_2 z_3 = 0 \; (E \cdot E = -2);$$
$$\tilde{E}_8 : z_1^2 + z_2^3 + z_3^6 + \lambda_3 z_1 z_2 z_3 = 0 \; (E \cdot E = -1);$$
$$\lambda_1^3 + 27 \neq 0, \lambda_2^4 + 64 \neq 0, \lambda - 432 \neq 0.$$

On the other hand, cusp singularities are sigularities appearing on the Satake compactifications of Hilbert modular surface. —— and the defining equation of (X,x) is given by the following form in some coordinates z_1, z_2, and z_3 around x:

$$T_{p,q,r} : z_1^p + z_2^q + z_3^r + \lambda z_1 z_2 z_3 = 0,$$
$$\frac{1}{p} + \frac{1}{q} + \frac{1}{r} < 1, \; \lambda \neq 0.$$

Anagolus to simple elliptic singularities, a three-dimensional Gorenstein purely elliptic singularity of $(0,2)$-type is called a simple $K3$ singularity which is characterized as a normal isolated singularity such that the exceptional set of any $\mathbf{Q}$-factorial terminal modification is a normal $K3$ surface by Ishii-Watanabe [3].

We are interested in hypersurface purely elliptic singularities. In general, for an n-dimensional purely elliptic singularity (X,x) defined by a nondegenerate holomorphic function f, the condition that (X,x) is of $(0,i)$-type can be stated by the geometric condition of the figure associated with with f in $\mathbf{R}^{n+1}$ called the Newton boundary $\Gamma(f)$ of f(see 1.4). Yonemura [10] and Fletcher [1] independently classified nondegenerate hypersurface simple $K3$ singularities into ninety-five classes in terms of weight of f and found nondegenerate quasi-homogenious polynomials for each weight.

In this note, we construct examples of polynomials defining a Gorenstein purely elliptic singularity of $(0,1)$-type at the origine of $\mathbf{C}^4$ from the weights of hypersurface simple $K3$ singularities listed by Yonemura and Fletcher in Section 2 and a result of good resolution of a hypersurface purely elliptic singularity of $(0,1)$-type in Section 3, and give results of resolution of singularity for two hypersurface purely elliptic singularities of $(0,1)$-type.

1 Preliminaries.

Definition 1.1 (Watanabe [8]) *Let (X, x) be a normal isolated singularity. For any positive integer m,*

$$\delta_m \stackrel{def}{=} dim_{\mathbf{C}}\Gamma(X - \{x\}, \mathcal{O}(mK))/L^{2/m}(X - \{x\}),$$

where K is the canonical line bundle on $X - \{x\}$, and $L^{2/m}(X - \{x\})$ is the set of all $L^{2/m}$-integrable (at x) holomorphic m-ple n-forms on $X - \{x\}$.

Definition 1.2 (Watanabe [8]) *A singularity (X, x) is said to be purely elliptic if $\delta_m = 1$ for every $m \in \mathbf{N}$.*

Definition 1.3 (Ishii [2]) *A quasi-Gorenstein singularity (X, x) is of $(0, i)$-type if $H^{n-1}(E_J, \mathcal{O}_E)$ consists of the $(0, i)$-Hodge component $H^{0,i}(E_J)$, where*

$$\mathbf{C} \simeq H^{n-1}(E_J, \mathcal{O}_E) = Gr_F^0 H^{n-1}(E_J) = \bigoplus_{i=0}^{n-1} H^{0,i}(E_J)$$

Assume that (X, x) is a hypersurface singularity defined by a nondegenerate polynomial $f = \sum a_\nu z^\nu \in \mathbf{C}[z_0, z_1, ..., z_n]$, and $x = O \in \mathbf{C}^{n+1}$.

The Newton boundary $\Gamma(f)$ of f is the union of the compact faces of $\Gamma_+(f)$, where $\Gamma_+(f)$ is the convex hull of $\cup_{a_\nu \neq 0}(\nu + R_{\geq 0}^{n+1})$ in R^{n+1}.

For any face Δ of $\Gamma_+(f)$, set $f_\Delta := \sum_{\nu \in \Delta} a_\nu z^\nu$.

We say f to be nondegenerate, if

$$\frac{\partial f_\Delta}{\partial z_0} = \frac{\partial f_\Delta}{\partial z_1} = \cdots = \frac{\partial f_\Delta}{\partial z_n} = 0$$

has no solution in $(\mathbf{C}^*)^{n+1}$ for any face Δ.

When f is nondegenerate, the condition for (X, x) to be a purely elliptic singularity of $(0, i)$-type is given as follows :

Theorem 1.4 (Watanabe [9]) *Let f be a nondegenerate polynomial and suppose $X = \{f = 0\}$ has an isolated singularity at $x = O \in \mathbf{C}^{n+1}$;*

(1) (X, x) is purely elliptic if and only if $(1, 1, \cdots, 1) \in \Gamma(f)$.

(2) Let $n = 3$ and let Δ_0 be the face of $\Gamma(f)$ containing the point $(1, 1, 1, 1)$ in the relative interior of Δ_0. Then (X, x) is a singularity of (0,i)-type if and only if $dim_{\mathbf{R}}\Delta_0 = i + 1$ for $i = 1$ and 2.

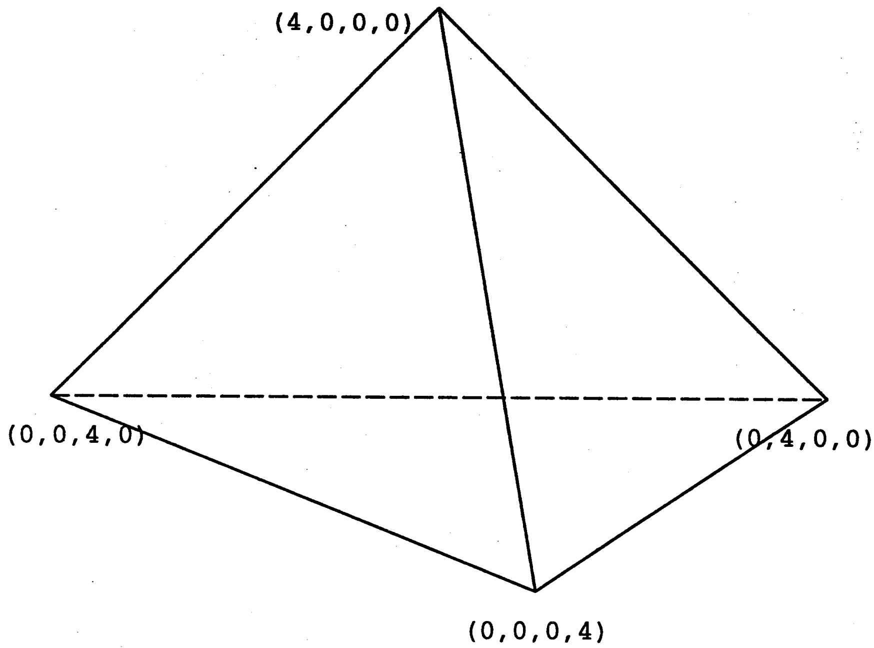

Figure 1: The Newton boundary of f .

Thus if f is nondegenerate and defines a simple $K3$ singularity, then f_{Δ_0} is a quasi-homogenious polynomial of a uniquely determined weight α called the *weight* of f and denoted $\alpha(f)$. Namely, $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) \in \mathbf{Q}^4_{\geq 0}$ and $deg_\alpha(\nu) := \sum_{i=1}^4 \alpha_i \nu_i = 1$ for any $\nu \in \Delta_0$. In paticular, $\sum_{i=1}^4 \alpha_i = 1$, since $(1,1,1,1)$ is always contained in Δ_0. Let $W' = \{\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) \in \mathbf{Q}^4_{\geq 0} | \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1\}$ and for an element α of W', set $T(\alpha) = \{\nu \in \mathbf{Z}^4_{\geq 0} | \alpha\nu = 1\}$ and $< T(\alpha) > := \{\sum_{\nu \in T(\alpha)} t_\nu \nu \in \mathbf{R}^4 | t_\nu \in \mathbf{R}_{\geq 0}\}$. Then the set $< T(\alpha) >$ is a closed cone in $\mathbf{R}^4$ spanned by $T(\alpha)$. Let $W_4 := \{\alpha \in W' | (1,1,1,1) \in Int < T(\alpha) >, \alpha_1 \geq \alpha_2 \geq \alpha_3 \geq \alpha_4\}$. By Theorem 1.4, W_4 is the set of weights of simple $K3$ singularities.

Proposition 1.5 (Yonemura [10], Fletcher [1]) *The cardinality of* W_4 *is* 95.

2 Method of construction

In this section, we construct examples of polynomials defining purely elliptic singularities of (0,1)-type. Our method of construction is due to Yonemura's classification of nondegenerate hypersurface simple $K3$ singularities in terms of the weight of a polynomial (see [10]). We illustlate the construction with one of the examples which we obtained :

Assume that $\alpha = (1,1,1,1)$ is a weight, which is the weight of the nondegenerate polynomial

$$f = X^4 + Y^4 + Z^4 + W^4 \ \in \ \mathbf{C}[X,Y,Z,W]$$

in Yonemura's ninety-five examples.

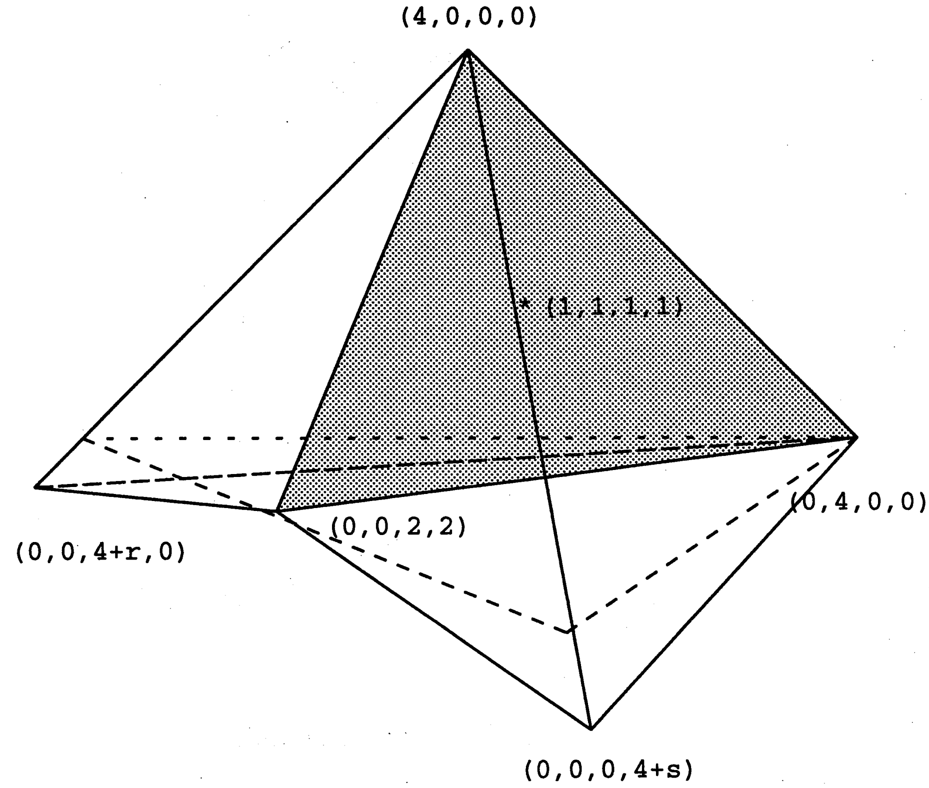

Figure 2: The Newton boundary of $g_{r,s}$.

The Newton boudary $\Gamma(f)$ of f is shown in Fig.1, where the tetrahedron is the intersection of $\mathbf{R}^4_{\geq 0}$ and the hyperplane $\{(x_1, x_2, x_3, x_4) \in \mathbf{R}^4 | x_1 + x_2 + x_3 + x_4 = 4\}$, and hence it is the 3-dimensional face of $\Gamma_+(f)$ containing $(1,1,1,1)$ in its relative interior. In this 3-dimensional face, there exist triangles containing $(1,1,1,1)$ in their relative interiors, with their vertices on the latices. For example, the triangle Δ_0 with vertices $(4,0,0,0), (0,4,0,0)$ and $(0,0,2,2)$ exists. This triangle Δ_0 is the intersection of two hyperplanes, i.e. the hyperplane through $(4,0,0,0), (0,4,0,0), (0,0,4+r,0)$ and $(0,0,2,2)$, and the hyperplane through $(4,0,0,0), (0,4,0,0), (0,0,0,4+s)$ and $(0,0,2,2)$, where $r \geq 0$, $s \geq 0$, $(r,s) \neq (0,0)$. On the other hand, consider the Newton boundary $\Gamma(g_{r,s})$ of $g_{r,s}$, where $g_{r,s} = X^4 + Y^4 + Z^{4+r} + W^{4+s} + Z^2W^2$. Then $\Gamma(g_{r,s})$ is the union of the tetrahedron with vertices $(4,0,0,0), (0,4,0,0), (0,0,4+r)$ and $(0,0,2,2)$, and the tetrahedron with vertices $(4,0,0,0), (0,4,0,0), (0,0,0,4+s)$ and $(0,0,2,2)$. In addition, Δ_0 is the 2-dimensional face of $\Gamma_+(g_{r,s})$ containing $(1,1,1,1)$ in its relative interior. Thus if $g_{r,s}$ is nondegenerate and $X = \{g_{r,s} = 0\} \subset \mathbf{C}^4$ has an isolated singularity only at the origin $x = O(\in \mathbf{C}^4)$, then x is a purely elliptic singularity of (0,1)-type by Theorem 1.4. In fact, $g_{1,1} = X^4 + Y^4 + Z^5 + W^5 + Z^2W^2$ and $g_{0,1} = X^4 + Y^4 + Z^4 + W^5 + Z^2W^2$ are nondegenerate and define isolated singularities only at the origin $x = O$.

In the same way, we could find 87 polynomials for 87 weights which appear in Yonemura's classification, which are listed in Table 5. But for five weights we could not construct any example, since the correspond tetrahedrons do not contain any triangle as

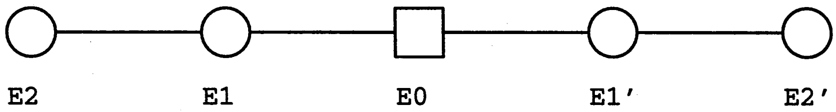

Figure 3: The dual graph of exceptional sets .

above. And we did not give examples for three weights which contain only triangles which seem to be irregular.

These examples are obtained from only one attempt for us to get examples in order to investigate purely elliptic singularities of (0,1)-type. We cannot say that it is sufficient to investigate these examples. In other words, there exists a polynomial which defines another purely elliptic singularity of (0,1)-type essentially different from our examples.

3 The results of resolution of a hypersurface purely elliptic singularity.

For the singularity (X,O) , $X=\{X^4+Y^4+Z^5+W^6+Z^2W^2\}$ which we gave in Section 2, we made a good resolution of singularity by using torus embeddings. The results are as follows.

Let $\pi:(\tilde{X},E)\longrightarrow(X,O)$ be the resolution of the singularity (X,O), where E is the exceptional set which has five irreducible components and whose dual graph is shown in Fig. 3. Let E_J be the essential devisor.

Then

$$E_J=E_1+E_0+E_1',$$

where E_0 is a surface birational to elliptic ruled surface and both E_1 and E_1' are rational ruled surfaces.

The intersection curves $C=E_0\cap E_1$ and $C'=E_0\cap E_1'$ are nonsingular elliptic curves.

The self-intersection numbers of C and C' in E_0 are -2, and those in E_1 and in E_1' are 8, respectively.

We give other results of the resolutions of the singularities of two examples by using torus embeddings in Table 3 and in Table 4. In each of the two cases, we obtained good resolutions of the singularities. In the tables, $E_{2,0}$ and $E_{3,0}$ are surfaces birational to elliptic ruled surfaces.

Table 3: The result of the resolution of the singularity obtaind as an example for the weight $(\frac{1}{3}, \frac{1}{3}, \frac{1}{6}, \frac{1}{6})$.

YN-3 : (X, o)	$X = \{g_2 = 0\}$, o : *the origin* $\in \mathbf{C}$
Polynomial	$g_2 = X^3 + Y^3 + Z^7 + W^7 + Z^3W^3$
The essential divisor E_J	$E_{2,1} + E_{2,0} + E'_{2,1}$
The intersection curves	$C = E_{2,1} \cap E_{2,0}, C' = E_{2,0} \cap E'_{2,1}$
The intersection numbers of C and C'	$C \cdot C = C' \cdot C' = -3$ in $E_{2,0}$; $C \cdot C = 9$ in $E_{2,1}$; $C' \cdot C' = 9$ in $E'_{2,1}$

Table 4: The result of the resolution of the singularity obtaind as an example for the weight $(\frac{1}{2}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$.

YN-5 : (X, o)	$X = \{g_3 = 0\}$, o : *the origin* $\in \mathbf{C}$
Polynomial	$g_2 = X^2 + Y^6 + Z^7 + W^7 + Z^3W^3$
The essential divisor E_J	$E_{3,1} + E_{3,0} + E'_{3,1}$
The intersection curves	$C = E_{3,1} \cap E_{3,0}, C' = E_{3,0} \cap E'_{3,1}$
The intersection numbers of C and C'	$C \cdot C = C' \cdot C' = -1$ in $E_{3,0}$; $C \cdot C = 6$ in $E_{3,1}$; $C' \cdot C' = 6$ in $E'_{3,1}$

Table 5: Examples of polynomials defining purely elliptic singularities of $(0,1)$-type.

The weights	The obtained polynomials
YN-1 : $\alpha=(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4})=(1,1,1,1;4)$	$g_s(t)=X^4+Y^4+Z^4+tW^4+W^{5+s}+Z^2W^2,(s\geq 0,t\in\mathbf{C})$
YN-2 : $\alpha=(\frac{1}{3},\frac{1}{4},\frac{1}{4},\frac{1}{6})=(4,3,3,2;12)$	$g_s(t)=X^3+Y^3Z+Z^4+tW^6+W^{7+s}+Z^2W^3,(s\geq 0,t\in\mathbf{C})$
YN-3 : $\alpha=(\frac{1}{3},\frac{1}{3},\frac{1}{6},\frac{1}{6})=(2,2,1,1;6)$	$g_s(t)=X^3+Y^3+Z^6+tW^6+W^{7+s}+Z^3W^3,(s\geq 0,t\in\mathbf{C})$ $g_r(u)=X^3+Y^3+uZ^6+Z^{7+r}+W^6+Z^3W^3,(r\geq 0,u\in\mathbf{C})$
*YN-4 : $\alpha=(\frac{1}{3},\frac{1}{3},\frac{1}{4},\frac{1}{12})=(4,4,3,1;12)$	$g_s(t)=X^3+Y^3+Z^4+tW^{12}+W^{13+s}+Z^3W^3,(s\geq 0,t\in\mathbf{C})$
YN-5 : $\alpha=(\frac{1}{2},\frac{1}{6},\frac{1}{6},\frac{1}{6})=(3,1,1,1;6)$	$g_s(t)=X^2+Y^6+Z^6+tW^6+W^{7+s}+Z^3W^3,(s\geq 0,t\in\mathbf{C})$ $g_r(u)=X^2+Y^6+uZ^6+Z^{7+r}+W^6+Z^3W^3,(r\geq 0,u\in\mathbf{C})$
YN-6 : $\alpha=(\frac{1}{2},\frac{1}{5},\frac{1}{5},\frac{1}{10})=(5,2,2,1;10)$	$g_s(t)=X^2+Y^4Z+Z^5+tW^{10}+W^{11+s}+Z^3W^4,(s\geq 0,t\in\mathbf{C})$
YN-7 : $\alpha=(\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{8})=(4,2,1,1;8)$	$g_s(t)=X^2+Y^4+Z^8+tW^8+W^{9+s}+Z^4W^4,(s\geq 0,t\in\mathbf{C})$ $g_r(u)=X^2+Y^4+uZ^8+Z^{9+r}+W^8+Z^4W^4,(r\geq 0,u\in\mathbf{C})$
*YN-8 : $\alpha=(\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{12})=(6,3,2,1;12)$	$g_s(t)=X^2+Y^4+Z^6+tW^{12}+W^{13+s}+Z^4W^4,(s\geq 0,t\in\mathbf{C})$
*YN-9 : $\alpha=(\frac{1}{2},\frac{1}{4},\frac{1}{5},\frac{1}{20})=(10,5,4,1;20)$	$g_s(t)=X^2+Y^4+Z^5+tW^{20}+W^{21+s}+Z^4W^4,(s\geq 0,t\in\mathbf{C})$
YN-10 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{1}{12},\frac{1}{12})=(6,4,1,1;12)$	$g_s(t)=X^2+Y^3+Z^{12}+tW^{12}+W^{13+s}+Z^6W^6,(s\geq 0,t\in\mathbf{C})$ $g_r(u)=X^2+Y^3+uZ^{12}+Z^{13+r}+W^{12}+Z^6W^6,(r\geq 0,u\in\mathbf{C})$
*YN-11 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{1}{10},\frac{1}{15})=(15,10,3,2;30)$	$g_s(t)=X^2+Y^3+Z^{10}+tW^{15}+W^{16+s}+Z^6W^6,(s\geq 0,t\in\mathbf{C})$
*YN-12 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{1}{9},\frac{1}{18})=(9,6,2,1;18)$	$g_s(t)=X^2+Y^3+Z^9+tW^{18}+W^{19+s}+Z^6W^6,(s\geq 0,t\in\mathbf{C})$
*YN-13 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{1}{8},\frac{1}{24})=(12,8,3,1;24)$	$g_s(t)=X^2+Y^3+Z^8+tW^{24}+W^{25+s}+Z^6W^6,(s\geq 0,t\in\mathbf{C})$
*YN-14 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{1}{7},\frac{1}{42})=(21,14,6,1;42)$	$g_s(t)=X^2+Y^3+Z^7+tW^{42}+W^{43+s}+Z^6W^6,(s\geq 0,t\in\mathbf{C})$
YN-15 : $\alpha=(\frac{1}{3},\frac{4}{15},\frac{1}{5},\frac{1}{5})=(5,4,3,3;15)$	$g_s(t)=X^3+Y^3Z+Z^5+tW^5+W^{6+s}+Z^2W^3,(s\geq 0,t\in\mathbf{C})$ $g_r(u)=X^3+Y^3Z+uZ^5+Z^{6+r}+W^5+Z^2W^3,(r\geq 0,u\in\mathbf{C})$ $h_s(t)=X^3+Y^3W+Z^5+tW^5+W^{6+s}+Z^3W^2,(s\geq 0,t\in\mathbf{C})$ $h_r(u)=X^3+Y^3W+uZ^5+Z^{6+r}+W^5+Z^3W^2,(r\geq 0,u\in\mathbf{C})$
YN-16 : $\alpha=(\frac{1}{3},\frac{7}{24},\frac{1}{4},\frac{1}{8})=(8,7,6,3;24)$	$g_s(t)=X^3+Y^3W+Z^4+tW^8+W^{9+s}+Z^3W^2,(s\geq 0,t\in\mathbf{C})$
YN-17 : $\alpha=(\frac{1}{3},\frac{1}{3},\frac{1}{5},\frac{2}{15})=(5,5,3,2;15)$	$g_s(t)=X^3+Y^3+Z^5+tZW^6+W^{8+s}+Z^3W^3,(s\geq 0,t\in\mathbf{C})$
YN-18 : $\alpha=(\frac{1}{3},\frac{1}{3},\frac{2}{9},\frac{1}{9})=(3,3,2,1;9)$	$g_s(t)=X^3+Y^3+Z^4W+tW^9+W^{10+s}+Z^3W^3,(s\geq 0,t\in\mathbf{C})$
YN-19 : $\alpha=(\frac{3}{8},\frac{1}{4},\frac{1}{4},\frac{1}{8})=(3,2,2,1;8)$	$g_s(t)=X^2Z+Y^4+Z^4+tW^8+W^{9+s}+Z^2W^4,(s\geq 0,t\in\mathbf{C})$
YN-20 : $\alpha=(\frac{3}{8},\frac{1}{3},\frac{1}{4},\frac{1}{24})=(9,8,6,1;24)$	$g_s(t)=X^2Z+Y^3+Z^4+tW^{24}+W^{25+s}+Z^3W^6,(s\geq 0,t\in\mathbf{C})$
YN-21 : $\alpha=(\frac{2}{5},\frac{1}{5},\frac{1}{5},\frac{1}{5})=(2,1,1,1;5)$	$g_s(t)=X^2Z+Y^4W+Z^5+tW^5+W^{6+s}+Z^2W^3,(s\geq 0,t\in\mathbf{C})$ $g_r(u)=X^2Z+Y^4W+uZ^5+Z^{6+r}+W^5+Z^2W^3,(r\geq 0,u\in\mathbf{C})$
YN-22 : $\alpha=(\frac{2}{5},\frac{1}{3},\frac{1}{5},\frac{1}{15})=(6,5,3,1;15)$	$g_s(t)=X^2Z+Y^3+Z^5+tW^{15}+W^{16+s}+Z^3W^6,(s\geq 0,t\in\mathbf{C})$
YN-23 : $\alpha=(\frac{5}{12},\frac{1}{4},\frac{1}{6},\frac{1}{6})=(5,3,2,2;12)$	$g_s(t)=X^2Z+Y^4+Z^6+tW^6+W^{7+s}+Z^2W^4,(s\geq 0,t\in\mathbf{C})$ $g_r(u)=X^2Z+Y^4+uZ^6+Z^{7+r}+W^6+Z^2W^4,(r\geq 0,u\in\mathbf{C})$ $h_s(t)=X^2W+Y^4+Z^6+tW^6+W^{7+s}+Z^4W^2,(s\geq 0,t\in\mathbf{C})$ $h_r(u)=X^2W+Y^4+uZ^6+Z^{7+r}+W^6+Z^4W^2,(r\geq 0,u\in\mathbf{C})$
YN-24 : $\alpha=(\frac{5}{12},\frac{1}{3},\frac{1}{6},\frac{1}{12})=(5,4,2,1;12)$	$g_s(t)=X^2Z+Y^3+Z^6+tW^{12}+W^{13+s}+Z^3W^6,(s\geq 0,t\in\mathbf{C})$
YN-25 : $\alpha=(\frac{4}{9},\frac{1}{3},\frac{1}{9},\frac{1}{9})=(4,3,1,1;9)$	$g_s(t)=X^2Z+Y^3+Z^9+tW^9+W^{10+s}+Z^3W^6,(s\geq 0,t\in\mathbf{C})$ $g_r(u)=X^2Z+Y^3+uZ^9+Z^{10+r}+W^9+Z^3W^6,(r\geq 0,u\in\mathbf{C})$ $h_s(t)=X^2W+Y^3+Z^9+tW^9+W^{10+s}+Z^6W^3,(s\geq 0,t\in\mathbf{C})$ $h_r(u)=X^2W+Y^3+uZ^9+Z^{10+r}+W^9+Z^6W^3,(r\geq 0,u\in\mathbf{C})$

YN-26 : $\alpha=(\frac{9}{20},\frac{1}{4},\frac{1}{5},\frac{1}{10})=(9,5,4,2;20)$	$g_s(t)=X^2W+Y^4+Z^5+tW^{10}+W^{11+s}+Z^4W^2,(s\geq 0,t\in\mathbf{C})$
YN-27 : $\alpha=(\frac{11}{24},\frac{1}{3},\frac{1}{8},\frac{1}{12})=(11,8,4,2;24)$	$g_s(t)=X^2W+Y^3+Z^8+tW^{12}+W^{13+s}+Z^6W^3,(s\geq 0,t\in\mathbf{C})$
YN-28 : $\alpha=(\frac{10}{21},\frac{1}{3},\frac{1}{7},\frac{1}{21})=(10,7,3,1;21)$	$g_s(t)=X^2W+Y^3+Z^7+tW^{21}+W^{22+s}+Z^6W^3,(s\geq 0,t\in\mathbf{C})$
YN-29 : $\alpha=(\frac{1}{2},\frac{1}{5},\frac{1}{6},\frac{2}{15})=(15,6,5,4;30)$	$g_s(t)=X^2+Y^5+Z^6+tYW^6+W^{8+s}+Y^3W^3,(s\geq 0,t\in\mathbf{C})$
YN-30 : $\alpha=(\frac{1}{2},\frac{1}{5},\frac{7}{40},\frac{1}{8})=(20,8,7,5;40)$	no triangle
YN-31 : $\alpha=(\frac{1}{2},\frac{5}{24},\frac{1}{6},\frac{1}{8})=(12,5,4,3;24)$	$g_s(t)=X^2+Y^4Z+Z^6+tW^8+W^{9+s}+Z^3W^4,(s\geq 0,t\in\mathbf{C})$
YN-32 : $\alpha=(\frac{1}{2},\frac{3}{14},\frac{1}{7},\frac{1}{7})=(7,3,2,2;14)$	$g_s(t)=X^2+Y^4Z+Z^7+tW^7+W^{8+s}+Z^3W^4,(s\geq 0,t\in\mathbf{C})$ $g_r(u)=X^2+Y^4Z+uZ^7+Z^{8+r}+W^7+Z^3W^4,(r\geq 0,u\in\mathbf{C})$ $h_s(t)=X^2+Y^4W+Z^7+tW^7+W^{8+s}+Z^4W^3,(s\geq 0,t\in\mathbf{C})$ $h_r(u)=X^2+Y^4W+uZ^7+Z^{8+r}+W^7+Z^4W^3,(r\geq 0,u\in\mathbf{C})$
YN-33 : $\alpha=(\frac{1}{2},\frac{2}{9},\frac{1}{6},\frac{1}{9})=(9,4,3,2;18)$	$g_s(t)=X^2+Y^4W+Z^6+tW^9+W^{10+s}+Z^4W^3,(s\geq 0,t\in\mathbf{C})$
YN-34 : $\alpha=(\frac{1}{2},\frac{7}{30},\frac{1}{5},\frac{1}{15})=(15,7,6,2;30)$	$g_s(t)=X^2+Y^4W+Z^5+tW^{15}+W^{16+s}+Z^4W^3,(s\geq 0,t\in\mathbf{C})$
YN-35 : $\alpha=(\frac{1}{2},\frac{1}{4},\frac{1}{7},\frac{3}{28})=(14,7,4,3;28)$	$g_s(t)=X^2+Y^4+Z^7+tZW^8+W^{10+s}+Z^4W^4,(s\geq 0,t\in\mathbf{C})$
YN-36 : $\alpha=(\frac{1}{2},\frac{1}{4},\frac{3}{20},\frac{1}{10})=(10,5,3,2;20)$	$g_s(t)=X^2+Y^4+Z^6W+tW^{10}+W^{11+s}+Z^4W^4,(s\geq 0,t\in\mathbf{C})$
YN-37 : $\alpha=(\frac{1}{2},\frac{1}{4},\frac{3}{16},\frac{1}{16})=(8,4,3,1;16)$	$g_s(t)=X^2+Y^4+Z^5W+tW^{16}+W^{17+s}+Z^4W^4,(s\geq 0,t\in\mathbf{C})$
YN-38 : $\alpha=(\frac{1}{2},\frac{4}{15},\frac{1}{5},\frac{1}{30})=(15,8,6,1;30)$	$g_s(t)=X^2+Y^3Z+Z^5+tW^{30}+W^{31+s}+Z^4W^6,(s\geq 0,t\in\mathbf{C})$
YN-39 : $\alpha=(\frac{1}{2},\frac{5}{18},\frac{1}{6},\frac{1}{18})=(9,5,3,1;18)$	$g_s(t)=X^2+Y^3Z+Z^6+tW^{18}+W^{19+s}+Z^4W^6,(s\geq 0,t\in\mathbf{C})$
YN-40 : $\alpha=(\frac{1}{2},\frac{2}{7},\frac{1}{7},\frac{1}{14})=(7,4,2,1;14)$	$g_s(t)=X^2+Y^3Z+Z^7+tW^{14}+W^{15+s}+Z^4W^6,(s\geq 0,t\in\mathbf{C})$
YN-41 : $\alpha=(\frac{1}{2},\frac{7}{24},\frac{1}{8},\frac{1}{12})=(12,7,3,2;24)$	$g_s(t)=X^2+Y^3Z+Z^8+tW^{12}+W^{13+s}+Z^4W^6,(s\geq 0,t\in\mathbf{C})$
YN-42 : $\alpha=(\frac{1}{2},\frac{3}{10},\frac{1}{10},\frac{1}{10})=(5,3,1,1;10)$	$g_s(t)=X^2+Y^3Z+Z^{10}+tW^{10}+W^{11+s}+Z^4W^6,(s\geq 0,t\in\mathbf{C})$
YN-43 : $\alpha=(\frac{1}{2},\frac{11}{36},\frac{1}{9},\frac{1}{12})=(18,11,4,3;36)$	$g_s(t)=X^2+Y^3W+Z^9+tW^{12}+W^{13+s}+Z^6W^4,(s\geq 0,t\in\mathbf{C})$
YN-44 : $\alpha=(\frac{1}{2},\frac{5}{16},\frac{1}{8},\frac{1}{16})=(8,5,2,1;16)$	$g_s(t)=X^2+Y^3W+Z^8+tW^{16}+W^{17+s}+Z^6W^4,(s\geq 0,t\in\mathbf{C})$
YN-45 : $\alpha=(\frac{1}{2},\frac{9}{28},\frac{1}{7},\frac{1}{28})=(14,9,4,1;28)$	$g_s(t)=X^2+Y^3W+Z^7+tW^{28}+W^{29+s}+Z^6W^4,(s\geq 0,t\in\mathbf{C})$
YN-46 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{1}{11},\frac{5}{66})=(33,22,6,5;66)$	$g_s(t)=X^2+Y^3+Z^{11}+tZW^{12}+W^{14+s}+Z^6W^6,(s\geq 0,t\in\mathbf{C})$
YN-47 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{2}{21},\frac{1}{14})=(21,14,4,3;42)$	$g_s(t)=X^2+Y^3+YZ^7+tW^{14}+W^{15+s}+Z^6W^6,(s\geq 0,t\in\mathbf{C})$
YN-48 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{5}{48},\frac{1}{16})=(24,16,5,3;48)$	$g_s(t)=X^2+Y^3+Z^9W+tW^{16}+W^{17+s}+Z^6W^6,(s\geq 0,t\in\mathbf{C})$
YN-49 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{5}{42},\frac{1}{21})=(21,14,5,2;42)$	$g_s(t)=X^2+Y^3+Z^8W+tW^{21}+W^{22+s}+Z^6W^6,(s\geq 0,t\in\mathbf{C})$
YN-50 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{2}{15},\frac{1}{30})=(15,10,4,1;30)$	$g_s(t)=X^2+Y^3+YZ^5+tW^{30}+W^{31+s}+Z^6W^6,(s\geq 0,t\in\mathbf{C})$
YN-51 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{5}{36},\frac{1}{36})=(18,12,5,1;36)$	$g_s(t)=X^2+Y^3+Z^7W+tW^{36}+W^{37+s}+Z^6W^6,(s\geq 0,t\in\mathbf{C})$
YN-52 : $\alpha=(\frac{1}{3},\frac{1}{4},\frac{2}{9},\frac{7}{36})=(12,9,8,7;36)$	no triangle
YN-53 : $\alpha=(\frac{1}{3},\frac{5}{18},\frac{2}{9},\frac{1}{6})=(6,5,4,3;18)$	$g_s(t)=X^3+Y^3W+XZ^3+tW^6+W^{7+s}+Z^3W^2,(s\geq 0,t\in\mathbf{C})$
YN-54 : $\alpha=(\frac{1}{3},\frac{2}{7},\frac{5}{21},\frac{1}{7})=(7,6,5,3;21)$	$g_s(t)=X^3+Y^3W+YZ^3+tW^7+W^{8+s}+Z^3W^2,(s\geq 0,t\in\mathbf{C})$
YN-55 : $\alpha=(\frac{7}{20},\frac{3}{10},\frac{1}{4},\frac{1}{10})=(7,6,5,2;20)$	$g_s(t)=X^2Y+Y^3W+Z^4+tW^{10}+W^{11+s}+Y^2W^4,(s\geq 0,t\in\mathbf{C})$
YN-56 : $\alpha=(\frac{11}{30},\frac{4}{15},\frac{1}{5},\frac{1}{6})=(11,8,6,5;30)$	no triangle
YN-57 : $\alpha=(\frac{3}{8},\frac{1}{4},\frac{5}{24},\frac{1}{6})=(9,6,5,4;24)$	$g_s(t)=X^2Y+Y^4+Z^4W+tW^6+W^{7+s}+Y^2W^3,(s\geq 0,t\in\mathbf{C})$
YN-58 : $\alpha=(\frac{3}{8},\frac{5}{16},\frac{1}{4},\frac{1}{16})=(6,5,4,1;16)$	$g_s(t)=X^2Z+Y^3W+Z^4+tW^{16}+W^{17+s}+Z^3W^4,(s\geq 0,t\in\mathbf{C})$
YN-59 : $\alpha=(\frac{8}{21},\frac{1}{3},\frac{5}{21},\frac{1}{21})=(8,7,5,1;21)$	$g_s(t)=X^2Z+Y^3+Z^4W+tW^{21}+W^{22+s}+Z^3W^6,(s\geq 0,t\in\mathbf{C})$
YN-60 : $\alpha=(\frac{7}{18},\frac{1}{3},\frac{2}{9},\frac{1}{18})=(7,6,4,1;18)$	$g_s(t)=X^2Z+Y^3+YZ^3+tW^{18}+W^{19+s}+Z^3W^6,(s\geq 0,t\in\mathbf{C})$

YN-61 : $\alpha=(\frac{11}{28},\frac{1}{4},\frac{3}{14},\frac{1}{7})=(11,7,6,4;28)$	$g_s(t)=X^2Z+Y^4+Z^4W+tW^7+W^{8+s}+Z^2W^4,(s\geq0,t\in\mathbf{C})$
YN-62 : $\alpha=(\frac{2}{5},\frac{1}{4},\frac{1}{5},\frac{3}{20})=(8,5,4,3;20)$	$g_s(t)=X^2Z+Y^4+Z^5+tYW^5+W^{7+s}+Z^2W^4,(s\geq0,t\in\mathbf{C})$
YN-63 : $\alpha=(\frac{2}{5},\frac{3}{10},\frac{1}{5},\frac{1}{10})=(4,3,2,1;10)$	$g_s(t)=X^2Z+Y^3W+Z^5+tW^{10}+W^{11+s}+Z^3W^4,(s\geq0,t\in\mathbf{C})$
YN-64 : $\alpha=(\frac{5}{12},\frac{7}{24},\frac{1}{6},\frac{1}{8})=(10,7,4,3;24)$	$g_s(t)=X^2Z+Y^3W+Z^6+tW^8+W^{9+s}+Z^3W^4,(s\geq0,t\in\mathbf{C})$
YN-65 : $\alpha=(\frac{14}{33},\frac{1}{3},\frac{5}{33},\frac{1}{11})=(14,11,5,3;33)$	$g_s(t)=X^2Z+Y^3+Z^6W+tW^{11}+W^{12+s}+Z^3W^6,(s\geq0,t\in\mathbf{C})$
YN-66 : $\alpha=(\frac{3}{7},\frac{2}{7},\frac{1}{7},\frac{1}{7})=(3,2,1,1;7)$	$g_s(t)=X^2Z+Y^3W+Z^7+tW^7+W^{8+s}+Z^3W^4,(s\geq0,t\in\mathbf{C})$ $g_r(u)=X^2Z+Y^3W+tZ^7+Z^{8+r}+W^7+Z^3W^4,(r\geq0,u\in\mathbf{C})$ $h_s(t)=X^2W+Y^3Z+Z^7+tW^7+W^{8+s}+Z^4W^3,(s\geq0,t\in\mathbf{C})$ $h_r(u)=X^2W+Y^3Z+uZ^7+Z^{8+r}+W^7+Z^4W^3,(r\geq0,u\in\mathbf{C})$
YN-67 : $\alpha=(\frac{3}{7},\frac{1}{3},\frac{1}{7},\frac{2}{21})=(9,7,3,2;21)$	$g_s(t)=X^2Z+Y^3+Z^7+tZW^9+W^{11+s}+Z^3W^6,(s\geq0,t\in\mathbf{C})$
YN-68 : $\alpha=(\frac{13}{30},\frac{1}{3},\frac{2}{15},\frac{1}{10})=(13,10,4,3;30)$	$g_s(t)=X^2Z+Y^3+YZ^5+tW^{10}+W^{11+s}+Z^3W^6,(s\geq0,t\in\mathbf{C})$
YN-69 : $\alpha=(\frac{7}{16},\frac{1}{4},\frac{3}{16},\frac{1}{8})=(7,4,3,2;16)$	$g_s(t)=X^2W+Y^4+YZ^4+tW^8+W^{9+s}+Z^4W^2,(s\geq0,t\in\mathbf{C})$
YN-70 : $\alpha=(\frac{4}{9},\frac{5}{18},\frac{1}{6},\frac{1}{9})=(8,5,3,2;18)$	$g_s(t)=X^2W+Y^3Z+Z^6+tW^9+W^{10+s}+Z^4W^3,(s\geq0,t\in\mathbf{C})$
YN-71 : $\alpha=(\frac{7}{15},\frac{4}{15},\frac{1}{5},\frac{1}{15})=(7,4,3,1;15)$	$g_s(t)=X^2W+Y^3Z+Z^5+tW^{15}+W^{16+s}+Z^4W^3,(s\geq0,t\in\mathbf{C})$
YN-72 : $\alpha=(\frac{7}{15},\frac{1}{3},\frac{2}{15},\frac{1}{15})=(7,5,2,1;15)$	$g_s(t)=X^2W+Y^3+YZ^5+tW^{15}+W^{16+s}+Z^6W^3,(s\geq0,t\in\mathbf{C})$
YN-73 : $\alpha=(\frac{1}{2},\frac{1}{5},\frac{4}{25},\frac{7}{50})=(25,10,8,7;50)$	no triangle
YN-74 : $\alpha=(\frac{1}{2},\frac{7}{32},\frac{5}{32},\frac{1}{8})=(16,7,5,4;32)$	$g_s(t)=X^2+Y^4W+YZ^5+tW^8+W^{9+s}+Z^4W^3,(s\geq0,t\in\mathbf{C})$
YN-75 : $\alpha=(\frac{1}{2},\frac{5}{22},\frac{2}{11},\frac{1}{11})=(11,5,4,2;22)$	?
YN-76 : $\alpha=(\frac{1}{2},\frac{3}{13},\frac{5}{26},\frac{1}{13})=(13,6,5,2;26)$	$g_s(t)=X^2+Y^4W+YZ^4+tW^{13}+W^{14+s}+Z^4W^3,(s\geq0,t\in\mathbf{C})$
YN-77 : $\alpha=(\frac{1}{2},\frac{7}{26},\frac{5}{26},\frac{1}{26})=(13,7,5,1;26)$	$g_s(t)=X^2+Y^3Z+Z^5W+tW^{26}+W^{27+s}+Z^4W^6,(s\geq0,t\in\mathbf{C})$
YN-78 : $\alpha=(\frac{1}{2},\frac{3}{11},\frac{2}{11},\frac{1}{22})=(11,6,4,1;22)$	$g_s(t)=X^2+Y^3Z+YZ^4+tW^{22}+W^{23+s}+Z^4W^6,(s\geq0,t\in\mathbf{C})$
YN-79 : $\alpha=(\frac{1}{2},\frac{9}{32},\frac{5}{32},\frac{1}{16})=(16,9,5,2;32)$	$g_s(t)=X^2+Y^3Z+Z^6W+tW^{16}+W^{17+s}+Z^4W^6,(s\geq0,t\in\mathbf{C})$
YN-80 : $\alpha=(\frac{1}{2},\frac{13}{44},\frac{5}{44},\frac{1}{11})=(22,13,5,4;44)$	$g_s(t)=X^2+Y^3Z+Z^8W+tW^{11}+W^{12+s}+Z^4W^6,(s\geq0,t\in\mathbf{C})$
YN-81 : $\alpha=(\frac{1}{2},\frac{4}{13},\frac{3}{26},\frac{1}{13})=(13,8,3,2;26)$	?
YN-82 : $\alpha=(\frac{1}{2},\frac{7}{22},\frac{3}{22},\frac{1}{22})=(11,7,3,1;22)$	$g_s(t)=X^2+Y^3W+YZ^5+tW^{22}+W^{23+s}+Z^6W^4,(s\geq0,t\in\mathbf{C})$
YN-83 : $\alpha=(\frac{1}{2},\frac{1}{3},\frac{5}{54},\frac{2}{27})=(27,18,5,4;54)$	$g_s(t)=X^2+Y^3+Z^{10}W+tYW^9+W^{14+s}+Z^6W^6,(s\geq0,t\in\mathbf{C})$
YN-84 : $\alpha=(\frac{1}{3},\frac{7}{27},\frac{2}{9},\frac{5}{27})=(9,7,6,5;27)$	$g_s(t)=X^3+Y^3Z+XZ^3+tYW^4+W^{6+s}+Z^2W^3,(s\geq0,t\in\mathbf{C})$
YN-85 : $\alpha=(\frac{5}{14},\frac{2}{7},\frac{3}{14},\frac{1}{7})=(5,4,3,2;14)$	?
YN-86 : $\alpha=(\frac{9}{25},\frac{7}{25},\frac{1}{5},\frac{4}{25})=(9,7,5,4;25)$	no triangle
YN-87 : $\alpha=(\frac{5}{13},\frac{4}{13},\frac{3}{13},\frac{1}{13})=(5,4,3,1;13)$	$g_s(t)=X^2Z+Y^3W+YZ^3+tW^{13}+W^{14+s}+Z^3W^4,(s\geq0,t\in\mathbf{C})$
YN-88 : $\alpha=(\frac{11}{27},\frac{1}{3},\frac{5}{27},\frac{2}{27})=(11,9,5,2;27)$	$g_s(t)=X^2Z+Y^3+Z^5W+tYW^9+W^{14+s}+Z^3W^6,(s\geq0,t\in\mathbf{C})$
YN-89 : $\alpha=(\frac{5}{11},\frac{3}{11},\frac{2}{11},\frac{1}{11})=(5,3,2,1;11)$	$g_s(t)=X^2W+Y^3Z+YZ^4+tW^{11}+W^{12+s}+Z^4W^3,(s\geq0,t\in\mathbf{C})$
YN-90 : $\alpha=(\frac{1}{2},\frac{7}{34},\frac{3}{17},\frac{2}{17})=(17,7,6,4;34)$	$g_s(t)=X^2+Y^4Z+Z^5W+tZW^7+W^{9+s}+Z^3W^4,(s\geq0,t\in\mathbf{C})$
YN-91 : $\alpha=(\frac{1}{2},\frac{4}{19},\frac{3}{19},\frac{5}{38})=(19,8,6,5;38)$	$g_s(t)=X^2+Y^4Z+YZ^5+tYW^6+W^{8+s}+Z^3W^4,(s\geq0,t\in\mathbf{C})$
YN-92 : $\alpha=(\frac{1}{2},\frac{11}{38},\frac{5}{38},\frac{3}{38})=(19,11,5,3;38)$	$g_s(t)=X^2+Y^3Z+Z^7W+tZW^{11}+W^{13+s}+Z^4W^6,(s\geq0,t\in\mathbf{C})$
YN-93 : $\alpha=(\frac{1}{2},\frac{5}{17},\frac{2}{17},\frac{3}{34})=(17,10,4,3;34)$	$g_s(t)=X^2+Y^3Z+YZ^6+tZW^{10}+W^{12+s}+Z^4W^6,(s\geq0,t\in\mathbf{C})$
YN-94 : $\alpha=(\frac{7}{19},\frac{5}{19},\frac{4}{19},\frac{3}{19})=(7,5,4,3;19)$	$g_s(t)=X^2Y+Y^3Z+Z^4W+tZW^5+W^{7+s}+Y^2W^3,(s\geq0,t\in\mathbf{C})$
YN-95 : $\alpha=(\frac{7}{17},\frac{5}{17},\frac{3}{17},\frac{2}{17})=(7,5,3,2;17)$	$g_s(t)=X^2Z+Y^3W+YZ^4+tZW^7+W^{9+s}+Z^3W^4,(s\geq0,t\in\mathbf{C})$

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